Fast Map Segmentation Method Based On Spectral Partition For Robot Semantic Navigation

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***Abstract*—Map segmentation method similar to the way human percept the external environment can decrease the computational complexity of robot navigation algorithm and SLAM problem. This paper presents an introduction to the application of spectral cluster method in map segmentation process. Then this paper presents several kinds of similarity measurement criteria to construct the similarity matrix. With these criteria, mobile robot can encountor different kinds of environments. Furthermore, this paper presents a self-adaptive clustering method based on silhouette coefficient criteria. As a result of that clustering result, an effective online segmentation method is prepared in this paper. Finally, the results of the experiment simulated on MobileSim platform demonstrate the performance of the proposal. The map segmentation method is which the high cohesion and low coupling are achieved in information. The segmentation results can greatly reduce the response time of such NP-Hard problems as large-scale SLAM and navigation problem.**

***Index Terms*—Spectral partition, Environment Segmentation, Mobile Robot, Adaptive Clustering**

I. Introduction

Simultaneous localization and mapping are a fundamental ability for autonomous robot.[1] With the growth of map size, the computational complexity of algorithm increases exponentially. L. Zhao and S. Huang presented a linear time complexity SLAM algorithm in [2] with the partition of the global map. And an algorithm is proposed in this paper to divide the global map into several connected sub-maps that decrease the complexity of SLAM and navigating algorithm.

In order to cope with large, complex environments, the internal representation acquired by the mobile robot can be organized as a hierarchy of maps which represent the entire environment at different levels of abstraction[3]. The Segmentation of the global map has two benefits:

1) Such segmentation can greatly reduce the computation complexity for large-scale SLAM and navigating algorithm [1].
2) Map division is a fundamental step for robot environment recognition.

In the field of robot environment recognition, it is important that computer vision systems be equipped with the means to summarize their contents in a natural language.[4] And a fundamental step of robot environment perception is to divide the global map into several connected sub-maps. Then a pattern recognition method labelling each sub-map into human recognition name, such as kitchen or bedroom and so on. In order to achieve harmonization of computer and human cognition.

Spectral clustering is an efficient computational technique based on a generalized eigenvalue problem[5], and it has been applied successfully in different areas. This algorithm is especially useful when the shape of the cluster is irregular. However, selecting features for global map and building similarity matrix are not a trivial task and its computational complexity is rather high [6]. Some methods are proposed in this paper can greatly reduce the computational complexity for spectral cluster algorithm. To measure the performance of clustering algorithm, a criterion based on silhouette coefficient is proposed in this paper.

The rest of the paper is organized as follows: After briefly introduce the process of Spectral Clustering method and presents a criterion for cluster algorithm based on silhouette coefficient in section 2, several different ways are proposed to choose representative features and build similarity matrix in section 3. This is a fundamental step for spectral clustering that highly affect the performance of the algorithm. Finally, in Section 4, an on-line approach is provided for map-partitioning, avoiding the non-linear time complexity.Experimental results are presented in section 5 demonstrating the efficiency and precision of the proposed method. In section 6, the main conclusions of this study and outline future research are drawn.

II. Map Partitioning algorithm based on Spectral theory

Spectral theory is a classical analysis and algebra method. It could transform the problem to be solved to a graph theory problem. Meanwhile, spectral cluster algorithm based on spectral theory is one of the most popular clustering algorithm at present. The situations where the distribution of given data is complicated and the cluster shape is irregular, spectral clustering algorithm could provide excellent clustering results.

The basic idea of spectral theory is taking research subject (each observation result) as a node of a weighted undirected graph. The similarity between research subjects is converted into the edges connecting each two nodes. Thus the original clustering problem is converted into the partition problem of graph.

The procedure of spectral partition algorithm is as follows:

1) Construct similarity matrix by the object differences
2) Construct normalization Laplacian matrix by similarity matrix
3) Calculate the smallest k eigenvalues and eigenvectors
4) Cluster the feature vectors

A. Construct Similarity Matrix

The first step of spectral partition is constructing similarity matrix, which requires to select a proper measurement to measure the similarity between two measurement results.

Define the similarity between i-th and j-th scanning result as S_{ij}. By calculating the similarity between all the scanning results, the similarity matrix S is obtained, defined as :

$$S = \begin{pmatrix} 0 & S_{12} & S_{13} & \cdots & S_{1n} \\ S_{12} & 0 & S_{23} & \cdots & S_{2n} \\ S_{13} & S_{23} & 0 & \cdots & S_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ S_{1n} & S_{2n} & S_{3n} & \cdots & S_{nn} \end{pmatrix} \tag{1}$$

There are different ways to measure the similarity between scanning results. In Section 3, a series of methods are presented to solve this problem, which can be applied in different areas.

It is oblivious that similarity metric proposed above has symmetry. I.e. $S_{ji} = S_{ij}$. By this fact the matrix building algorithm can reduce the amount of computation halfway. Nevertheless, the time complexity of similarity algorithm is $O(n^2)$. No doubt that such algorithm is unacceptable in a large global map. The following two reasons ensure the value for this algorithm. And in a certain measure, avoid the $O(n^2)$ time complexity:

- Not every scanning results taken by a mobile robot are used in the process of similar matrix building. Scanning frequency of laser range finder can reach more than 20 times per second. However, most of the results are very similar. In fact, the segmentation algorithm only accept scanning results when the change of position and attitude reached a given threshold. In this way, the approach greatly reduced the amount of computation.
- The partition result produced in off-line algorithm is a ”seed” for online algorithm. In the first moment of exploring, the mobile robot wandering a part of given area to train the cluster classifier. Then the on-line algorithm uses an adaptive instance set classified the new scanning result, dynamically determine whether the new result belongs to a new area. So that approach avoids iterates over the whole scanning results and decreases the time complexity into the linear time complexity.

B. Build Laplacian Matrix

By the similarity matrix, undirected weighted graph G is constructed as a graph of all scanning results. Each result is regarded as a vertex of this graph. And the weight of edge of the graph presents the similarity of results. Then calculate the Laplacian matrix of graph G , symbol as$L(G)$ for spectral clustering. An easy way to calculate $L(G)$ is described as follows:

$$L(G) = D(G) - S(G) \tag{2}$$

Parameter $S(G)$ in the above equation is the similarity matrix of G. And $D(G)$ is the degree matrix of graph G. It can be defined as follows:

$$D(G) = \begin{pmatrix} S_1 & 0 & 0 & 0 \\ 0 & S_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & S_n \end{pmatrix} \tag{3}$$

$S_1, S_2, \cdots S_n$ in the above equation presents the degree of graph G.

In practice, the graph G has too much edge. To decrease the amount of computation, the similarity matrix s si calculated with the above process, p is a pre given parameter:

$$switch = S_{min} + p(S_{max} - S_{min}) \tag{4}$$

$$S_{ij} = \begin{cases} 0 & s_{ij} \leqslant switch \\ S_{ij} & s_{ij} > switch \end{cases} \tag{5}$$

C. Normalized Cut of Graph

To get an normalized cut of graph G. Firstly normalized the Laplacian matrix of G[7].

$$L^{sym}_{i,j} = \begin{cases} 1 & i = j, deg(v_i) \neq 0 \\ -\dfrac{1}{\sqrt{deg(v_i)deg(v_j)}} & i \neq j, S_{ij} \neq 0 \\ 0 & otherwise \end{cases} \tag{6}$$

If the amount of cluster is given as k, that means the graph of total observation result are divided into k submaps. Then the smallest eigenvalue of $L^{sym}_{i,j}$ is found as the corresponding eigenvalue. The K-Means clustering result of those eigenvalue is the result of normalized cut.

D. Adaptive Cluster Method Based on Silhouette Coefficient Metric

In the approach mentioned above, the number of cluster center is given. In Practice, the value of k should be determined by the algorithm. A easy way to get the value of k is to guess the approximate value k', the try to cluster scanning results with $\cdots k'-2, k'-1, k', k'+1, k'+2, \cdots$ and see which value is better.

In order to evaluate the performance of cluster algorithm. Silhouette coefficient is introduced to evaluate the cluster result with cohesion and separation factors. With this criterion , the performance of different clustering strategy can be evaluated.

Define the clustering result in sample space X as a partition of X. I.e. $cluster(X) = \{x_1, x_2, \cdots, x_n\}$, has the following properties:

$$\begin{cases} X_1, X_2, \cdots, X_n \subset X \\ X_1 \cup X_2 \cup X_3 \cdots \cup X_m = S \\ X_i \cap X_j = \phi \qquad \forall i \neq j \end{cases} \tag{7}$$

For each cluster X_i in the above clustering result and each sample $x_j \in X_i$. The cohesion factor a_i is the average distance between every other sample in the cluster X_i. And the separation factor b_i is the minimum average distance between x_j and every other sample not in cluster X_i. Defined as follows:

$$a_i = \frac{\sum_{m=1}^{|X_i|} S(x_j, x_m)}{|X_j|} \quad j \neq i \tag{8}$$

$$b_i = \min\left(\frac{\sum_{m=1}^{|X_i|} S(x_j, x_m)}{|X_j|}\right) \quad j \neq i \tag{9}$$

For the sample x_j, silhouette coefficient is:

$$s_i = \frac{(b_i - a_i)}{max(a_i, b_i)} \tag{10}$$

And the silhouette coefficient for the clustering result is the average of every sample's silhouette coefficient.

Obviously, if $s_i < 0$, then the average distance between samples in cluster X_i are less than the nearest cluster. I.e. the clustering result is bad. If a_i tends to zero, or b_i is big enough, then s_i tends to 1. That means the result is good. Take the result with maximum silhouette coefficient as the result of adaptive cluster algorithm.

Consequence of K-Means clustering algorithm depend on the initial value of the serious problem. To achieve a more stable result, we run this algorithm many times and pick the result with maximum silhouette coefficient. This method is quick enough for mobile robot because the amount and dimension of sample are limited.

III. Similarity Creteria for Observation Results

The first step of spectral partition is constructing similarity matrix, which requires to select a proper measurement to measure the similarity between two measurement results. A series methods are given below , which can be applied in different situations:

- Match different scanning results by K-Nearest Neighbour (KNN) algorithm. Calculate the similarity of scanning results by measuring the sum of distances between obstacle points matched in that scanning results.
- Measure the similarity of two scanning results by Hausdroff distance.
- Extract feature points (usually corner point) from each scanning results. Judge the similarity of two scanning results by the quantity of shared characteristic points in two scanning results.

In this paper, laser range finder is used to construct the environment model. Figure 1 shows an observation result taken by a laser range finder. Point A indicates the position of robot's laser range finder. Dark lines in the figure present obstacles in the global map. The shadowed area is scanning area. Laser rangefinder starts from the left side of the boundary area, scans the given area with an interval angle $\Delta\theta$.

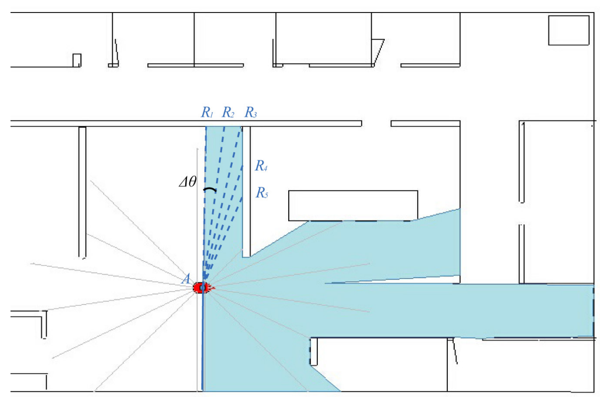

Fig. 1. A scanning result taken by robot laser sensor, Black lines presents the global map already given before experiments. Dark Area in the above figure is the scanning area.

The form of laser sensor data we used in this experiment can be acquired as follows:

$$readings' = [(x1, y1), (x2, y2), \cdots (xn, yn)] \tag{11}$$

In the above figure 1, scanning range is in 180 degrees front of robot. And the scanning precision is 1 degree. Each observation result contains 181 data points.

A. Nearest neighbour criteria

The scanning result similarity problem can be defined as follows:

In two dimensional space with Euclidean metric, given two points sets $Readings_a, Readings_b$, abbreviated as r_a, r_b.

Each points in r_a and r_b expressed as symbol r_ai, r_bi construct a mapping $f(r_b) \rightarrow r_a$. The similarity criteria $S(r_a, r_b)$ is defined as follows:

$$S(r_a, r_b) = \sum_{r_{bi} \in r_b} \|f(r_{bi}) - r_{bi}\| \tag{12}$$

For any $r_{bi} \in r_b$, $f(r_{bi})$ can minimize the value $S(r_a, r_b)$.

The essence of the Nearest Neighbour algorithm is to find an optimal match between the two scanning results(for example, r_a and r_b). The use of this algorithm considers the following factors:

1) Scanning data of laser sensor have good continuity. In general, adjacent scanning results are considerably the same. In some special cases (for example, the robot go through a door or a corner), scanning result has a sudden change. Through the perception of this change map segmentation mission can be expected to be completed.
2) The size of each scanning result is stable (for PeopleBot-sh, numbers of point in each scanning result is 181),which avoid the NearestNeighbors mismatch phenomenon by the asymmetry of sample size.
3) Although the time complexity of NearestNeighbors algorithm is $O_(n^2)$, the size of each laser scanning result is stable and small enough for the algorithm to produce a good response time.

Fig 2 shows the result of clustering algorithm when a robot wanders in a room and doorway. The highest peak of subset B represents the scanning result when the robot goes through the door. Other peaks come from the fast rotation of the robot when wandering in the room.

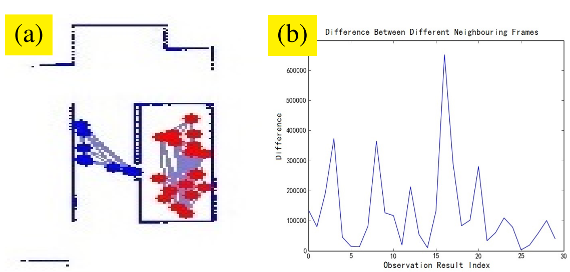

Fig. 2. Subset a shows the result of cluster algorithm when a robot wander in a room and doorway. Subset b shows the value of $S(r_{i-1}, r_i)$ when robot achieve the observation i

B. Hausdroff distance criteria

Another commonly used measurement methods is Hausdroff distance. The main idea of Hausdroff distance is to measure the maximum mismatch of two point sets. This metric is defined as follows:

$$\begin{cases} H(r_a, r_b) = \max(h(r_a, r_b), h(r_b, r_a)) \\ h(r_a, r_b) = \max(r_{ai}) \min(r_{bi}) \|r_{ai} - r_{bi}\| \\ h(r_b, r_a) = \max(r_{bi}) \min(r_{ai}) \|r_{ai} - r_{bi}\| \end{cases} \tag{13}$$

Symbol $h(r_a, r_b)$ and $h(r_b, r_a)$ is called unilateral Hausdorff distance. As $h(r_a, r_b)$ and $h(r_b, r_a)$ does not conform to the principle of symmetry. Hausdroff distance is defined as the maximum one of unilateral Hausdorff distance. Compared to equation 13, an improved hausdorff distance calculation algorithm called Rucklidge algorithm decreased the amount of computation. Hausdroff distance metric is simple and easy to parallelization and is a good choice when the size of scanning result is too large to calculate.

C. Feature point criteria

It is a simple and fast way to measure the similarity of different observation results. In the scanning process. Robot saves the feature point (usually a corner point) of the scanning result instead the whole scanning result. In this way the algorithm greatly reduced the space complexity.

In practice, the same feature point observed in different scanning result may have a deviation. Fig 3 presents such deviation. So the algorithm maintained a list $L = [(x_1, y_1), (x_2, y_2), \cdots (x_n, y_n)]$ to save every accepted feature point. Then define the acceptance range δ. To each observed feature point p_i, if there exists $p_{li} \in L$ makes $\|p_i - p_{li}\| < \delta$, then saves the point p_{li} instead p_i. In that way, each scanning result X_i can be saved as a list of feature point's index.

To measure the similarity between two scanning result X_i and X_j, the similarity factor S_{ij} is defined as follows:

$$S_{ij} = card(\{x | if x \in X_i and x \in X_j\}) \tag{14}$$

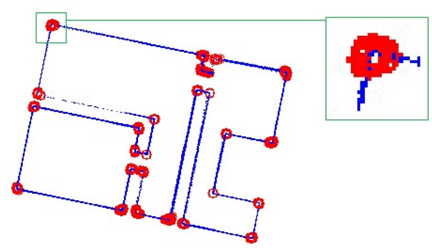

Fig. 3. Feature points observed in a map. Red circle in this figure presents the location of the feature point. And blue pixel presents obstacle point in the scanning result. Area in the green rectangle presents the measure deviation.

Fig 4 presents the similarity matrix constructed by different criteria when a robot is wandering in the room anticlockwise. We normalized the matrix to prevent the confusion by different matrices. Nearest Neighbour criteria and Hausdroff distance criterion presents $1 - S_i$ instead S_i so different similarity function increases monotonically. Because those observation results listed by time ,those observation result has a good consistency. So the diagonal part of such matrix is similar to partitioned matrices. Each partitioned part presents a room. The upper left area and the bottom right area present the same room because the robot return to the start point.

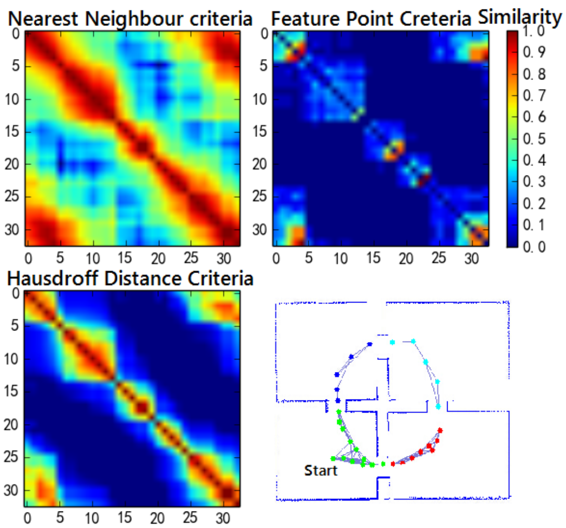

Fig. 4. Similarity matrix constructed by different criteria. Colour in the above figure presents the normalized S_ij value.

IV. Online Map Partitioning Method

To the autonomous mobile robot exploring the unknown area, map partitioning algorithm should determine the category of each scanning result dynamically. And in such situation, the amount of scanning result will have too much to calculate. Fig 5 presents an online map segmentation method to avoid that problem. In that way, mobile robot uses the offline partitioning result as a classifier. When the robot gets a new observation result, the algorithm determines that if such result belongs to an existing cluster or a new cluster. Then calculates the new position of the cluster center. If the cluster result is too much to calculate, some small adjacent cluster will be combined.

The online approach is as follows and shown in 6:

1) Autonomous mobile robot wanders in the room and explores part of the global map.
2) Archive separation threshold T with offline map partitioning method. T is the maximum similarity between each cluster.
3) For a new scanning result Xi, calculate the similarity S_{ij} between X_i and every other cluster center Z_j. Determine the category of that result with S_{ij} as follow:
   - If $S_{ij} \leq \theta T$ ($0 \leq \theta \leq 1$ is a pre given parameter determined by the complexity of map). Then X_i and Z_j belongs to same category.
   - If $S_{ij} \geq \theta T$, Then X_i and Z_j do not belongs to the same category.
   - If $\theta T \leq S_{ij} \leq T$ t, Then do not consider the belonging of X_i.
4) If for each center Z_j, $S_{ij} \geq T$, Then such scanning result belongs to a new cluster and take that result as a cluster center.
5) If X_i and Z_i belongs to the same category and the amount of scanner result in Z_i is t, then set cluster center with the equation below:

$$Z_i(t+1) = \frac{1}{t+1}[tZ_i(t) + X_i] \qquad (15)$$

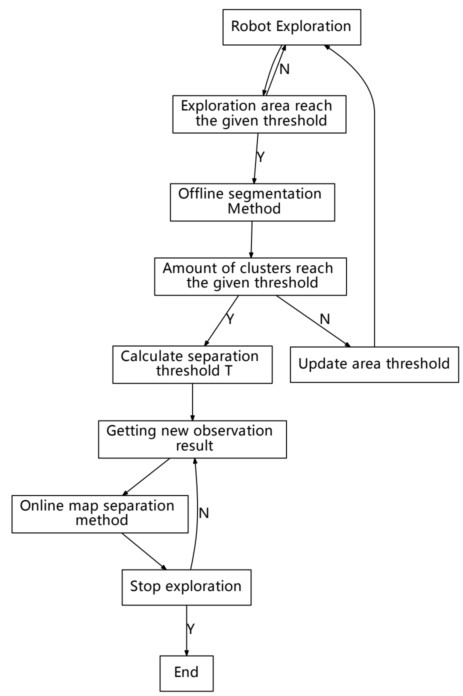

Fig. 5. Flowchart of online partitioning method.

V. Experimental Results

Fig 7 shows the result of map partitioning algorithm in a simple map shown as subset a . Algorithm run on an Intel I7 platform and is coded with Python. Runtime of this algorithm is 117 ms and can be further optimized.

Subset b, c, d shows the cluster result with different amount of the cluster center. The dark line is shown in the subset a is obstacle, and the blue contour in subset b, c, d comes from the scanning result. Each pixel presents an obstacle point scanned by laser range finder. Points in subset b, c, d shows

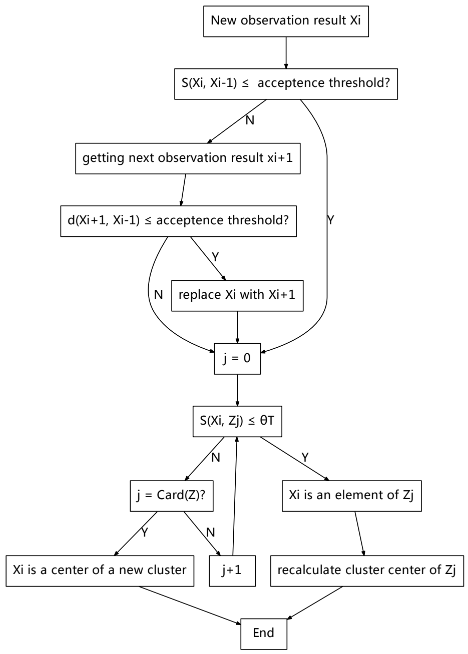

Fig. 6. A flow diagram presents the adaptive clustering method.

the position while the robot took a scan and colour of the point represents the cluster it belongs to. Edge between points presents the similarity between scanning results.

Silhouette coefficient of different cluster result presents in 7. From the result we know the maximum silhouette coefficient comes from $k = 3$ result in subset C.

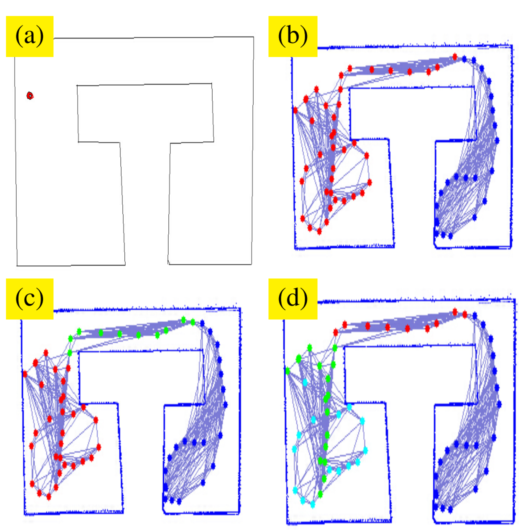

Fig. 7. Map partition result when amount of cluster center changes from 2 to 4. Subset a shows the original map. subset b, c, d is the result when value k is 2, 3, 4.

k	S_i
2	0.525564
3	0.709461
4	0.572336
5	0.548360
6	0.560596
7	0.537543

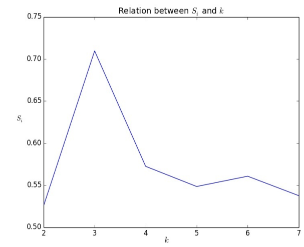

Fig. 8. The relationship between silhouette coefficient S_i and amount of cluster center k

From the results mentioned above, the performance of algorithm works well when the number of clusters is 2 or 3. When the cluster number gets to 4, the algorithm goes over partition. One reason is the amount of data is too small to combine the unique cluster. I.e the cohesion factor is not large enough.

Fig 9 shows an partition result based on feature point. If the environment is easy, algorithm based on feature point is greatly faster than algorithm based on KNN or Hausdroff distance. Because feature oriented map partitioning algorithm only involves setting operation and avoid evaluating eigenvalue and eigenvector. In this experiment, the amount of shared corner point is a criterion to measure the similarity between observation results.

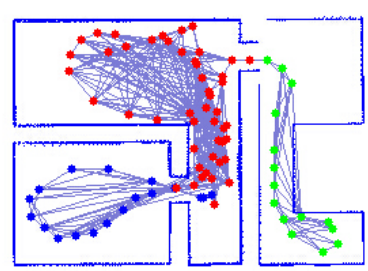

Fig. 9. A partition result in a more complex map.

In most of the conditions, the environment is messy. In that case, similarity criteria based on corner point or line don't work well. Meanwhile, In an irregular environment, criteria based on KNN algorithm and feature point work well. Fig 10 gives an example when robot wandering in a messy area. In this experiment KNN algorithm is used to measure the similarity between observation results.

Fig 11 shows a participation result comes from real environment. The map was built by a peoplebot-sh mobile robot and Hokuyo UST-10LX laser range finder. The environment

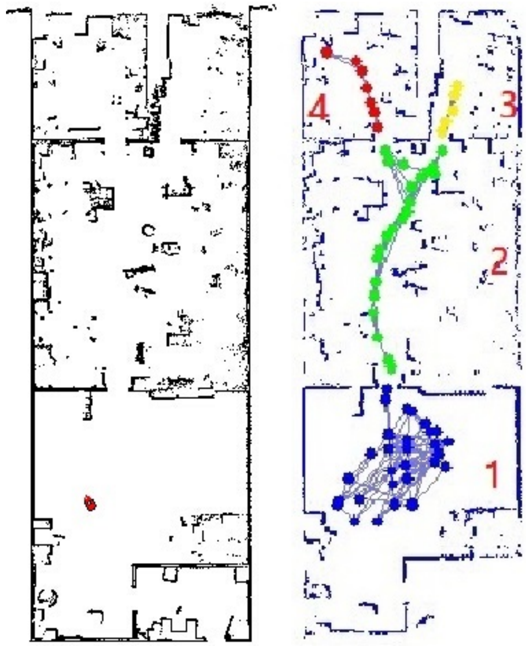

Fig. 10. A partition result in a more complex map.Left subset shows the global map and right shows the partition result

comes from 3rd floor of State Key Laboratory of Robotics and System, Harbin Insitute of Technology. According to the complexity of the environment, KNN similarity criteria was tested on this map. Red rectangle presents the participation result. The algorithm participated the map into six submaps. These submaps stand for the hall, aisle, two gaps between cubices and a seprate room. The sagmentation result is similar to the human cognition of environments. Different submaps share she minimum information in common.

Fig. 11. Algorithm test on a real envionment

VI. Conclusions and Future Work

In this paper, an efficient approach for map partitioning based on spectral cluster algorithm has been used on the data sequence achieved by laser range finder. To build Laplacian matrix , Four different way are given to measure the similarity between cluster results. Then we presents silhouette coefficient criteria to measure the performance of cluster algorithm. Finally, an online version of map partitioning algorithm decreased the time complexity. Experiments with both regular map and irregular environment presents its correctness and efficiency if the given information is enough to make the partition.

Future works will focused on the parallelization of map partitioning problem and the hierarchical navigation algorithm. Associated similarity measurement criteria will be further tested to adapt different kind of environment. Additional studies with machine learning algorithm and semantics knowledge will archive the harmonization of computer and human cognition.

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